Scaling in laminar natural convection in laterally heated cavities: Is turbulence essential in the classical scaling of heat transfer?

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We analyze heat transfer and flow properties in laminar natural convection driven by a horizontal temperature gradient in a closed cavity and propose that for the classical scaling of heat transfer turbulence does not play a decisive role. Direct numerical simulations were performed with the Rayleigh number (Ra) from 1 to 10^8 and the Prandtl number Pr=0.71. In the laminar steady flow regime with the Ra approximately from 10^3 to 10^7, power-law scalings of heat transfer and maximum velocity with Ra have exponents of 0.31 and 0.54, respectively. The scalings agree well with results obtained in turbulent Rayleigh-Bernard convection, turbulent convection in laterally heated cavities and laminar convection in inclined enclosures, etc., which, with some simple physical arguments and reviews of the literature, leads us to propose that turbulence is not essential for the classical near 1/3 power-law scaling of Nu.

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Natural convection in a gravitational field is ubiquitous in nature and in many important technological systems. It has been the subject of extensive studies in theory, experiments, and numerical simulations (cf., reviews [1–3] and references therein). Rayleigh-Bernard convection (RBC), a special form of natural convection driven by a vertical temperature gradient, has become a classic system for the study of buoyancy-induced turbulence. One of the most dramatic discoveries from classical experiments on this system in the turbulent convection regime is the (effective) power-law dependencies of heat transport on dimensionless buoyancy forcing with scaling exponents between 0.25 and 0.33 [4]. Significant efforts, both numerically and experimentally, have been directed at investigating the mechanisms and detailed scaling behavior of turbulent RBC. Another type of natural convection driven by a horizontal temperature gradient is just as important in practical applications but has received much less attention from the physics community. It is mostly selected as a validation problem to compare numerical algorithms designed for solving the Navier-Stokes or Boltzmann equations, or for turbulence modeling and computation. Theoretical, numerical, and experimental work [5–11] on this system has investigated flow patterns, temperature distributions, flow instabilities, etc., mostly focusing more on the transition to unsteady flow or the effects of aspect ratio on heat transfer at moderately high Rayleigh number (Ra).

We demonstrate that natural convection in these two configurations as well as plate [12] and inclined enclosures [13,14] share some important characteristics. Although the flow regimes can be very different in that one can be completely laminar whereas the other governed, at least in the bulk, by strong turbulent fluctuations, the heat transfer scaling with forcing is very similar within a certain range.

Natural convection is characterized by the Rayleigh number $\mathrm{Ra}=g\alpha\Delta T d^3/\nu\kappa$ and the Prandtl number $\mathrm{Pr}=\nu/\kappa$ with g the acceleration of gravity, α the thermal expansion coefficient, ΔT the applied temperature difference, d the distance along the temperature gradient, and ν and κ the kinematic viscosity and thermal diffusivity, respectively. The global response to buoyant forcing is measured by Nusselt number (Nu), mean (or max) flow velocity u, and Reynolds number. The last is for turbulent convection only.

In this study, we numerically investigate in detail power-law scaling (PLS) of Nu on Ra in laminar convection subject to horizontal temperature gradient. The classical scaling is nearly identical to that in turbulent RBC as well as in natural convection over single plate and in inclined enclosures. Based on detailed examinations we find that the existence of a large-scale circulation (LSC), the resultant boundary layers, and interior temperature distribution are sufficient to produce the classic near 1/3 PLS (here "near 1/3" means the exponent can be in the range from 0.286 to 0.33 as extensively reported in literature). We thus conjecture that (1) although turbulence produces rich nonuniversal flow dynamics it has little effect on the average global heat transfer. (2) Similar near 1/3 PLS is a universal characteristic for thermal convection and should exist in different flow regimes in all closed cavities with various temperature gradient arrangements. (3) Side-heated and inclined cavities may provide alternative routes to study the effects of turbulence on heat transfer in natural convection.

Mathematically, natural convection is generally described using the Boussinesq approximation which assumes that all fluid properties remain constant except in the buoyant force where the fluid density ρ is linearly proportional to the temperature. Consider a flow confined in a cavity with height H (z direction) and width L (x direction) where the aspect ratio of the cavity is defined as $\Gamma=H/L$. A dimensionless scaling for the flow uses width L, velocity κ/L, pressure $\rho\kappa^2\,\mathrm{Ra}/L^2$, time L^2/κ, and temperature difference ΔT $(=T_h-T_l)$ to nondimensionalize the respective quantities in the Boussinesq equations. Here T_h and T_l are the heated and cooled wall temperatures and ρ is the mean fluid density. The dimensionless Boussinesq equations for incompressible flow read [1]

$$\frac{\partial\vec{v}}{\partial t}+\vec{v}\cdot\vec{\nabla}\vec{v}=-\vec{\nabla}p+\nu_d\nabla^2\vec{v}+\mathrm{Ra}\,\mathrm{Pr}\,\Theta\hat{\mathbf{z}}, \qquad (1)$$

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$$\frac{\partial\Theta}{\partial t} + \vec{v} \cdot \vec{\nabla}\Theta = \kappa_d \nabla^2 \Theta, \qquad (2)$$

where $\vec{v}$, Θ, $\nu_d(=\mathrm{Pr}/\sqrt{\mathrm{Ra}})$, and $\kappa_d(=1/\sqrt{\mathrm{Ra}})$ are dimensionless velocity, temperature, viscosity, and thermal diffusivity, respectively. The last term on the right-hand side in Eq. (1) reflects the buoyant force.

In this paper, we present numerical results on laterally heated cavity convective flow in a relatively low but wide Ra range from 1 to 10^8 where PLS is observed. Focus is on the transition characteristics of laminar convection. First, we identify two flow stages in the laminar flow regime through the Ra dependence of heat transfer and flow properties. Then we compute the effective exponents in PLSs of velocity and Nu on Ra in the laminar flow regime in the moderately low Ra range from 10^4 to 10^7. We also compute the scalings of both the viscous boundary layer (VBL) and the thermal boundary layer (TBL), which are defined similarly in turbulent RBC. The existence of such boundary layers allows us to construct a scaling argument based on mixing-length theory and obtain the 1/3 PLS with a prefactor determined by a threshold Ra_t, similar to using the critical Ra_c in turbulent RBC.

We employ the lattice Boltzmann method [15] and perform simulations of the dimensionless Boussinesq equations in a square two-dimensional cavity ($\Gamma=1$) with Pr=0.71. The boundary conditions are hot ($\Theta_h=0.5$) at the left wall, cold ($\Theta_l=-0.5$) at the right wall, and adiabatic at the top and bottom boundaries. All four boundaries are nonslip. Initially, we set $\vec{v}=\Theta=0$ everywhere within the cavity so that the flow is driven only by the buoyant force. The computation scheme is described and validated elsewhere [16].

Two important properties for convective flow are Nu, which measures the enhancement of heat transfer by convection over conduction, and v_{max}, the maximum velocity amplitude in the field. The former is computed by $\mathrm{Nu}=\int_0^1 (u\Theta - \frac{\partial\Theta}{\partial x})|_x dz$ where the first term is the contribution from heat convection and the second is from heat conduction. By definition Nu=1 for conduction. We compute Nu at $x=0.5$ although any value of x within the width from 0 to 1 gives the same Nu.

Stationary temperature contours at different Ra are shown in Fig. 1. At low Ra (=10), the temperature gradient is distributed nearly uniformly over the whole field in the horizontal direction. Convection is weak and conduction dominates heat transfer. As Ra increases to about 10^3, buoyant forces become stronger and convection starts to play a role. The temperature distribution is deformed, and boundary layers begin to form along both sides. As Ra increases to 10^5, boundary layers are well-developed and the fluid becomes thermally stratified. At this stage, heat transfer within the thin boundary layers is dominated by conduction and by convection outside. For $\mathrm{Ra} \gtrsim 10^7$, the flow is further stratified and the boundary layers become very thin.

Quantitative measurements yield the dependence of Nu and v_{max} on Ra, see Figs. 2(a) and 2(b). Two stages in the laminar flow regime are captured. At low Ra, Nu remains approximately unity indicating that conduction dominates.

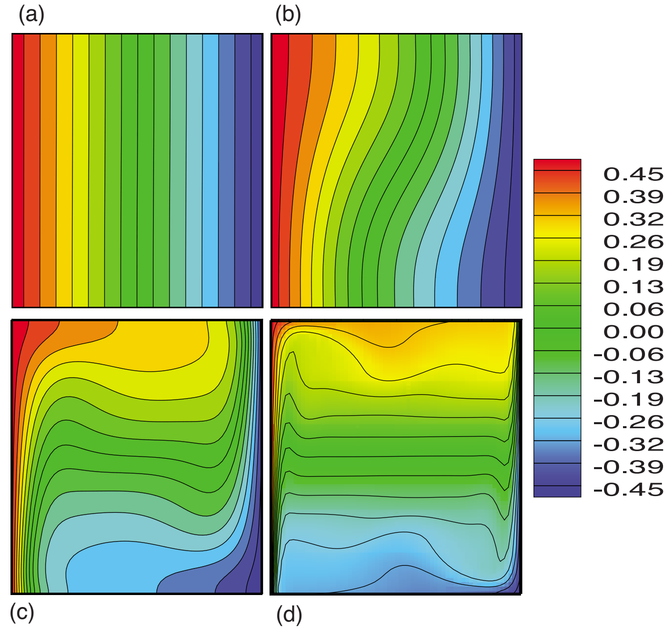

FIG. 1. (Color online) Stationary temperature contours at Ra numbers (a) 10, (b) 10^3, (c) 10^5, and (d) 10^7. Color coded temperature scale is shown on the right.

The growth of v_{max} is linear in Ra. In the second stage, both v_{max} and Nu exhibit power-law growth. The velocity magnitude scales as $v_{max} \sim 0.14\,\mathrm{Ra}^{0.54}$, close to the experimental measurement [17] for turbulent RBC where the scaling exponent for the velocity is 0.49. The dependence of Nu on Ra,

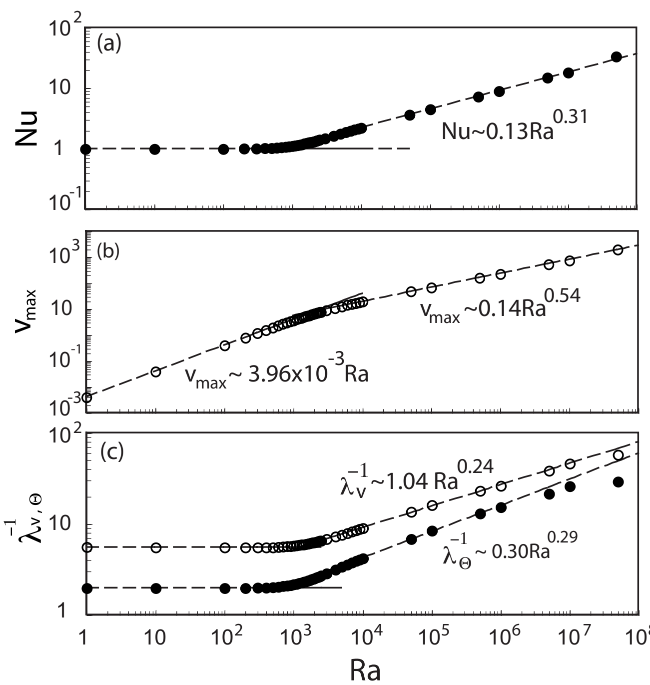

FIG. 2. Power-law scalings as a function of Ra. (a) Heat transfer Nu; (b) maximum velocity magnitude v_{max}; and (c) mean inverse boundary layer thickness for velocity (dots) and temperature (circles).

$\mathrm{Nu} \sim 0.13\,\mathrm{Ra}^{0.31}$, shows excellent correspondence with turbulent RBC results with a similar scaling exponent. This scaling exponent is consistent with previous numerical results [18] but is interpreted here in a different way.

The dependency of Nu on Γ is computed in the range of $\Gamma=0.5-20$ at Ra=5000. Nu monotonically decreases from 5 at $\Gamma=0.5$ to 2.4 at $\Gamma=20$, in qualitative agreement with the classical experimental data [19].

As mentioned above, the PLS ($\mathrm{Nu} \sim \mathrm{Ra}^{\beta}$) of a large Ra number RBC has been well-investigated. Experimental data reveal power-law dependencies of $\beta=0.25-0.33$, see Table 1 of Ref. [1]. Generally in turbulent RBC, mixing length theories predict $\mathrm{Nu} \sim \mathrm{Ra}^{1/3}$ for $\mathrm{Pr} \gtrsim 0.1$ [1] by supposing that the heat conduction is confined to the regions near the heated (or cooled) plates and that the two boundary layers do not communicate. From more general considerations, one gets a complicated diagram of regions with different PLSs for Nu between 0.25 and 0.5 [4] depending on the Ra and Pr numbers. There may not be a pure PLS for experiments that cross over from one region to another [20]. In [21], Niemela *et al.* reported the experimental measurement of $\mathrm{Nu}=0.124\,\mathrm{Ra}^{0.309}$ in the broadest range of Ra ($10^{6}-10^{17}$) using cryogenic helium gas near its critical point. This is remarkably close to the numerical result here ($0.13\,\mathrm{Ra}^{0.31}$) over a wide range of Ra.

The same PLS is found in inclined enclosures. It is reported that in the range of $2500 < \mathrm{Ra}\cos\phi < 10^{6}$, $\mathrm{Nu}=0.157(\mathrm{Ra}\cos\phi)^{0.29}$ with ϕ the incline angle of the device from the horizontal plane [13]. At higher Ra numbers when LSC forms and the boundary layers become independent, $\mathrm{Nu}=0.056(\mathrm{Ra}\cos\phi)^{0.33}$ is found to fit very closely all available experimental data [14]. In the single plate case [12], $\mathrm{Nu}=0.047\,\mathrm{Ra}^{0.33}$ best fits the experiment data with Ra from 10^{12} to 10^{14}. In these cases, the near 1/3 PLS exists in the flow regimes of either laminar and turbulent. There is no discontinuity in the transition.

We further characterize the boundary layer thickness development in the laminar convection flow regime. We define the x location of maximum vertical velocity magnitude (w) as the VBL thickness $\lambda_v(z)$ and the x location of the intersect of the slope of temperature gradient at the hot wall with the corresponding stratified temperature line as the TBL thickness $\lambda_\Theta(z) = (\Theta_h - \Theta|_{(0.5,z)}) / \frac{\partial\Theta}{\partial x}|_{(0,z)}$.

Figure 2(c) shows the mean inverse VBL and TBL thicknesses computed as $\lambda_v^{-1}=\Sigma_z[1/\lambda_v(z)]$ and $\lambda_\Theta^{-1}=\Sigma_z[1/\lambda_\Theta(z)]$ for different Ra. First, $\lambda_v < \lambda_\Theta$ for all Ra's, which is true for the $\mathrm{Pr}<1$ case. Second, according to the analysis above, after the fluid becomes stratified, heat transfer near the side boundaries is dominated by conduction. Therefore the growth of Nu should be roughly proportional to λ_Θ^{-1}. Comparing $\mathrm{Nu} \sim 0.13\,\mathrm{Ra}^{0.31}$ in Fig. 2(a) with $\lambda_\Theta^{-1} \sim 0.30\,\mathrm{Ra}^{0.29}$ in Fig. 2(c) we confirm this prediction ($\lambda_\Theta^{-1} \sim 2\,\mathrm{Nu}$ since the total TBL thickness is $2\lambda_\Theta$). It is seen that Nu grows slightly faster than λ_Θ^{-1}. This is because that convection still makes a small contribution in the boundary layers. Last and most importantly, the demonstration of boundary layer effects on the flow in different Ra ranges in turn helps in understanding the flow physics. For low Ra, the VBL and the TBL are not meaningful. The computed TBL is essentially half of the cavity width. After the flow starts to stratify, the VBL and the TBL form and begin to dominate heat transfer. The reduction of the boundary layer thickness follows a power law. As Ra reaches about 5×10^{7}, the flow becomes time dependent and perhaps turbulent, and the computation of the thickness of TBL and VBL can no longer utilize the same approach. Experimental measurement of thermal boundary layer thickness in turbulence gas convection driven by sidewall heating, defined as the position at which the temperature rms is maximum, shows the scaling with Ra to have an exponent of 0.29 in the range of Ra from 5×10^{5} to 10^{11} ($\lambda_\theta^{-1}=0.28\,\mathrm{Ra}^{0.29}$) [22], which is in agreement with the numerical results here. This is identical to that found in RBC with a slightly larger prefactor [23], which we believe is from symmetry-induced suppression of LSC. Again, there is no discontinuity in scaling of boundary layer thickness from laminar to turbulent flows.

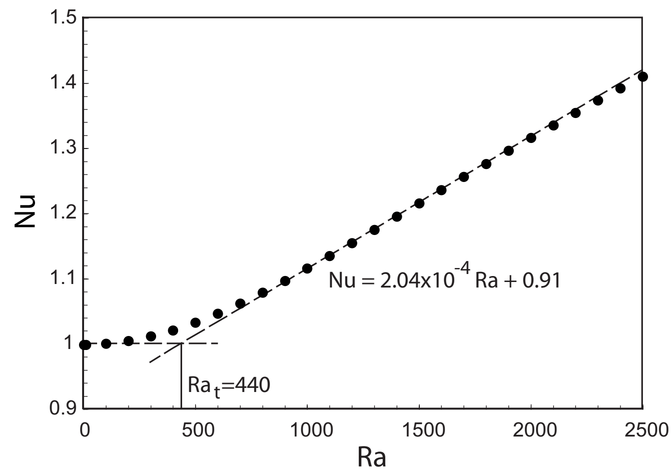

FIG. 3. Threshold Ra_t through extrapolation of the linear region to the base value of conduction, Nu=1.

Figure 3 zooms in on the growth of Nu at very low Ra ($<10^{3}$). After a short rounded region where $\mathrm{Nu} \sim \mathrm{Ra}^{2}$, Nu becomes linear in Ra. These two regions were predicted by Batchelor in 1954 [24]. We extract a threshold Rayleigh number Ra_t analogous to the critical Ra_c in RBC through the extrapolation of the linearity to the base value of Nu=1 for pure conduction. This threshold Ra_t characterizes the onset of significant convection influence on the heat transfer. It is found that Ra_t is nearly independent of the cavity aspect ratio.

In what follows, we perform an analysis [25] analogous to RBC to interpret the near 1/3 PLS of Nu. For $\mathrm{Ra}<\mathrm{Ra}_t$, see Figs. 1 and 2, buoyancy induced flow is very weak and heat transfer is dominated by conduction. When Ra exceeds Ra_t, convection starts to dominate the heat transfer. Large scale shear flow forms as Ra reaches around 10^{5}. At this stage, the core region is stratified with large convective flow transporting heat, and there is effectively no temperature gradient horizontally. All significant horizontal temperature gradients are in two boundary layers with a total thickness $\lambda_t(=2\lambda_\Theta)$. The conduction dominated boundary layer thickness may be determined by $\mathrm{Ra}_t=(g\alpha\Delta T\lambda_t^{3})/(\nu\kappa)$ whereas for the cavity $\mathrm{Ra}=(g\alpha\Delta TL^{3})/(\nu\kappa)$. Since the heat flux $j_h=\kappa\Delta T/\lambda_t$ and conduction flux $j_c = \kappa \Delta T / L$, by definition we have $\mathrm{Nu} = j_h / j_c = L / \lambda_t = (\mathrm{Ra}/\mathrm{Ra}_t)^{1/3} = 0.13\,\mathrm{Ra}^{1/3}$. This result slightly over predicts Nu obtained from the simulations. Following the same simple procedure for RBC, and not using the marginal stability argument for the boundary layer thickness, $\mathrm{Nu} = (\mathrm{Ra}/\mathrm{Ra}_c)^{1/3} = 0.084\,\mathrm{Ra}^{1/3}$. In addition, the natural existence of a LSC in the laterally heated cavities and the attainment of 0.31 PLS nearly identical to that in RBC suggests the modification of LSC to the 1/3 PLS may have some universal features.

This work systematically examines the heat transfer and flow properties in laminar natural convection in laterally heated cavities with numerical simulations and demonstrates the fundamental role of LSC in natural convection in closed cavities. The transition from conduction-dominated heat transfer to a convection-dominated regime, first proposed in the theoretical work of Batchelor, is clearly analyzed. The resulting threshold Ra_t, akin to the role of the critical Ra_c in RBC, is explored and used in the study of the transition to power scaling in Nu, velocity, and thermal and viscous boundary layers. Such scalings, all existing in laminar flow, are found to be nearly identical to those in turbulent RBC and other natural convection systems, prompting us to propose that LSC and the resultant boundary layers and stratified interior are sufficient to produce the classical near 1/3 (0.31) PLS for heat transfer in all natural convection in closed cavities over an extended range, and that the role of turbulence is not essential in this regard. Our claim is bolstered by the very suggestive derivation of the prefactor which means that heat transfer scaling is solely based on laminar flow properties.

Some previous studies came very close but did not reach this explicit conclusion. Further evidence can be gleamed from studies on the so-called "ultimate state," where turbulence truly dominates (the exponent is 1/2). In experimental [26] and theoretical [27] studies, the onset of such scaling does *not* coincide with the onset of turbulence. Rather, breakdown of boundary layers appears to be necessary in reaching the ultimate state, and it takes place deep into the hard turbulence regime.

Based on this work and literature review, we propose a set of experiments that can validate our conjectures. In RBC systems, one can change the cross section to change the symmetry, thus changing the persistence and fluctuation of the LSC. A simultaneous measurement of Nu, LSC amplitude, and turbulence intensity should show strong correlation between the first two, but weak correlation with the last. One can also tilt the convection systems and measure the scaling of heat transfer as the flow transitions from laminar to turbulent. If it is the LSC, not turbulence, that determines the heat transfer, the near 1/3 scaling would span the transition with no discontinuity. This is clearly seen in Belmonte's work [22,23] for boundary layer thickness. Further confirmation may be achieved in heat transfer measurement, and extension of Ra to beyond 10^{11}. In addition, one can suppress turbulence in RBC and repeat the heat transfer measurement.

In summary, we believe that based on our work and insight from a large body of work, the complexity and mechanisms for the "near 1/3" scaling in Nu vs Ra in natural convection in cavities (RBC and side-heated are special limiting cases) would be reduced and elucidated, respectively, by our conjecture. This work establishes a paradigm in the study of natural convection in closed cavities and offers alternative routes to study the effects of LSC and turbulence.

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